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This is a linear homogeneous ode and can be solved using standard methods.

Let Y(s)=L[y(t)](s). Instead of solving directly for y(t), we derive a   
new equation for Y(s). The first step is to take the Laplace transform of both sides of the  
original differential equation. We have

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the Laplace transform of the function 0 is 0. If we look at  
the left-hand side, we have

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Here we have used the fact that y(0)=2. And,

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Hence, we have

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The Laplace-transformed differential equation is

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Solving for Y(s), we have

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the method of partial fractions:

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Using linearity of the inverse transform, we have

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Consider the ode:

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This problem has an inhomogeneous term. For this problem the particular solution can be determined   
using variation of parameters or the method of undetermined   
coefficients. Using the Laplace transform technique we can solve for   
the homogeneous and particular solutions at the same time.

Let Y(s) be the Laplace transform of y(t). Taking the Laplace transform   
of the differential equation we have:

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The Laplace transform of the LHS  L[y''+4y'+5y] is

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The Laplace transform of the RHS is

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Equating the LHS and RHS and using the fact that y(0)=1 y'(0)=2, we   
obtain

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Solving for Y(s), we obtain:

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Using the method of partial fractions

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Using the fact that the inverse of 1/(s-1) is e^t and that the inverse of   
1/[(s+2)^2+1] is exp(2t)sin(t), it follows that

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